Aspects of Cosmology : Mathematics and Physics
Ecole Polytechnique
September 25-27th 2024

Schedule

	Wednesday	Thursday	Friday
9h30	Welcome coffee		
9h45	Welcome presentation	Welcome coffee	Welcome coffee
10h	Julien Larena	David Fajman	Gemma Hood
11h	Break	Break	Break
11h30	Jan Sbierski	David Andriot	Leonardo Senatore
12h30	Lunch	Lunch	Lunch
14h	Giulia Cusin	Léo Bigorgne	Volker Schlue
15h	Break	Break	Break
15h30	Nikos Athanasiou	Stéphane Colombi	
16h30	Scott Melville		

Abstracts :

David Andriot : Attempts on de Sitter and quintessence from string theory

In this talk, I will present attempts to provide an origin to dark energy from string theory, together with associated mathematical tools and difficulties. I will start with the search and properties of de Sitter solutions in 10-dimensional supergravity, then present a study of exponential quintessence and its cosmological solutions, and finally show some analytic results on quintessence solutions during radiation-matter domination.

Nikolaos Athanasiou : A localized construction of Kasner-like singularities

Ever since the discovery of the first explicit solutions to the Einstein equations containing a Big Bang singularity, there have been attempts to understand the nature of the general cosmological singularity. Kasner-like singularities are a specific class of spacelike singularities whose leading order behavior toward the singularity resembles that of a Kasner solution at each spatial point on the singular hypersurface. While in the so-called sub-critical regime, Kasner-like behavior at the Big Bang singularity is expected to be generic, the celebrated BKL heuristics imply that, outside this regime, the general Big-Bang singularity is expected to be oscillatory and Kasner-like behaviour to be non-generic.
In this talk, we will present a localization of the construction of Kasner-like singularities in (1+3)-vacuum that was established by Fournodavlos-Luk. We work with a first-order symmetric hyperbolic fomulation of the Einstein equations, at first order in the second fundamental form of the Gaussian time slices. We will also mention some possible applications of this construction to black holes and Big Bang singularities. This is joint work with Grigoris Fournodavlos.

Léo Bigorgne : Late-time asymptotics of small data solutions for the Vlasov-Poisson system

We first review recent works concerning modified scattering for the small data solutions to the Vlasov-Poisson system. In particular, we will see how to identify the logarithmic corrections of the linear characteristics along which the distribution function converges to a scattering state.
Then, we will show that one can subsequently identify a hierarchy of asymptotic conservation laws for the distribution function, from which we derive polyhomogeneous expansions for the spatial density up to any finite order. This also provides higher order corrections to the linear characteristics, allowing us to improve the rate of convergence of the distribution function to its scattering state.

Stéphane Colombi : Some numerical aspects of the Vlasov-Poisson equations

In the concordance model of the formation of large-scale structures in the Universe, the distribution of matter is dominated by a dark component that can be approximated by a self-gravitating, collisionless fluid, the dynamics of which is described by the Vlasov-Poisson equations. I will study some properties of these equations and how they are solved numerically with the traditional N-body method and direct solvers using ``semi-Lagrangian'' methods and sophisticated computational geometry techniques. Focusing here on the dynamical evolution of single objects, I will study what happens for various systems, evolving from an initially warm or initially cold state. The concept of mean field limit will be approached through comparisons between N-body and Vlasov codes that will show that it is sometimes difficult, if not nearly impossible,
to disentangle numerical effects from physical effects.

Giulia Cusin : Gravitational wave backgrounds: from theoretical modelling to detection prospects

When looking at a population of astrophysical gravitational wave sources we can either decide to focus on those sources that are particularly bright, and build a catalogue, or characterise collectively the superposition of all signals from all sources from the onset of stellar activity until today. This stochastic background of gravitational radiation is an interesting observable as it can allow us to extract astrophysical and cosmological information that cannot be extracted from the study of individual events and other cosmological probes. In this talk I will review the state of the art of background theoretical modeling, illustrate the potential of its cross-correlation with cosmological observables, and present future detection prospects.

David Fajman : A stability phase transition for cosmological Fluids

Relativistic fluids that evolve in expanding cosmological spacetimes experience a damping effect due to the spacetime’s expansion. This damping effect reduces the tendency of the fluid to form shocks during its evolution. A second parameter that affects the regularity of the fluid as it evolves is its speed of sound.
A fast speed of sound increases the tendency of the fluid to form shocks. We are interested in scenarios where a fluid is unstable, meaning that arbitrary small perturbations of a homogeneous fluid evolve to a shock in finite time. In the parameter space spanned by speed of sound of the fluid and expansion rate of spacetime there are regions of instability and those of stability. At their

interface phase transitions between stable and unstable behaviour occur. We demonstrate a complete picture of this parameter space locating the phase transition by complementary analytical and numerical techniques. From the perspectives of cosmology this contributes to an understanding of conditions in the Universe's evolution where shocks form from homogeneously distributed matter and conditions where in contrast matter remains regular.

Gemma Hood : A scattering construction for nonlinear wave equations on Kerr-Anti de Sitter spacetimes

Given the sharp logarithmic decay of linear waves on the Kerr-AdS black hole (Holzegel, Smulevici '13), it is expected that the Kerr-AdS spacetime is unstable as a solution of the Einstein vacuum equations. However, the scattering construction presented here for exponentially decaying nonlinear waves on a fixed Kerr-AdS background serves as a first step to confronting the scattering problem for the full Einstein system. In this context, one may hope to derive a class of perturbations of Kerr-AdS which remain 'close' and dissipate sufficiently fast.

Julien Larena : Gravitational lensing in arbitrary spacetime

After a general discussion on the nature of gravitational lensing, we will review various methods and approximations to describe a set of lenses within an arbitrary spacetime background.

Finally, we will apply these approximations to two complementary cosmological problems: the description of small, but finite beams of light, and that of isolated strong lenses in an inhomogeneous universe.

Scott Melville : Positive structures in cosmology

The scattering of low-energy particles is described by a complex function - the scattering amplitude - which possesses a number of non-trivial mathematical properties that can be connected to positive geometries. These positivity properties have been useful in searching for traces of new particles in colliders. This talk will describe how these structures are modified in cosmology and how they can be connected with observational data.

Jan Sbierski : Singularity structure of FLRW spacetimes at low regularities

This talk investigates the structure of the Big Bang singularity in a variety of FLRW spacetimes. It is straightforward to compute scalar curvature invariants to determine whether a curvature singularity is present which excludes a continuation as a strong solution to the Einstein equations. In this talk the focus is on capturing the singularity structure at the level of the connection and the metric itself, determining which geometric quantities blow up and in which regularity class the solution breaks down.

Volker Schlue : Expanding black hole cosmologies: On the non-linear stability of Kerr de Sitter spacetimes

The Kerr de Sitter geometry models a rotating black hole in an expanding universe. I will review its stability properties in the context of the Einstein vacuum equations with positive cosmological constant, and present recent progress on the non-linear stability problem for the cosmological region. Among others, the talk describes contributions by H Friedrich, P Hintz and A Vasy, and my recent joint work with G Fournodavlos.

Leonardo Senatore : Exploring how inflation starts out of inhomogeneous initial conditions.

We show how mathematical techniques such as mean curvature flow and the Thurston Geometrization Classification (Poincare' hypothesis) can be used to shed light on how inflation starts.